Introduction to Causal Discovery

More on (T)PC in practice: Tiers and finite sample problems

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separation in graph $\Leftrightarrow$ (conditional) independence

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Practical issue: Algorithm sensitive to statistical errors.

With background knowledge: Estimate restricted equivalence class represented by an MPDAG [Perković et al., 2017].

$\Rightarrow$ Contains information additional to independence.

$\Rightarrow$ Estimate more robust to statistical errors [Petersen et al., 2021, Bang et al., 2024].

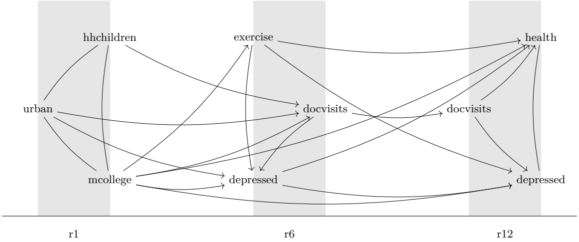
Informativeness

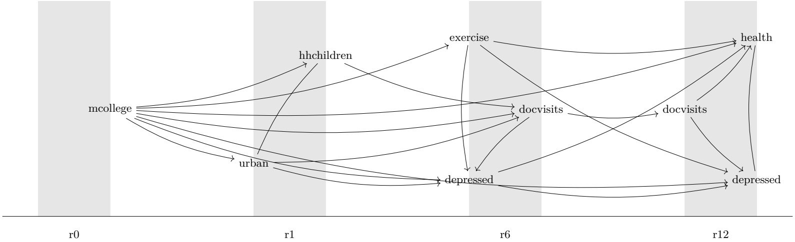
CPDAGs: Encode (conditional) independencies

MPDAGs: Encode (conditional) independencies and additional causal information

DAGs: Encode (conditional) independencies and all causal information

Informativeness





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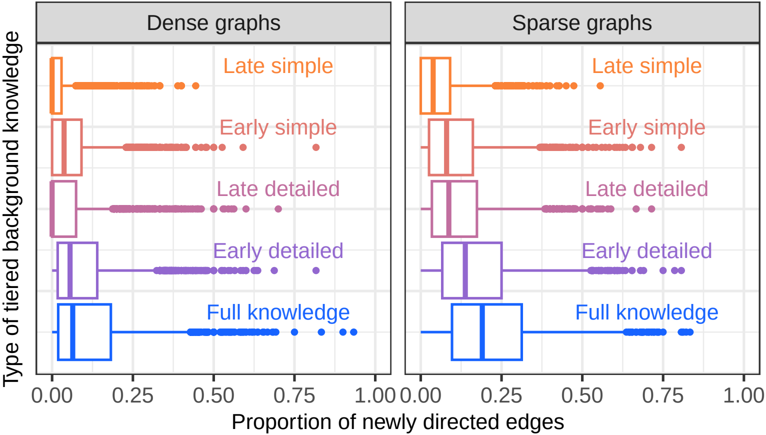
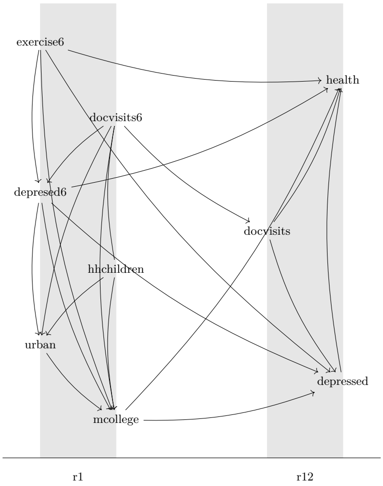
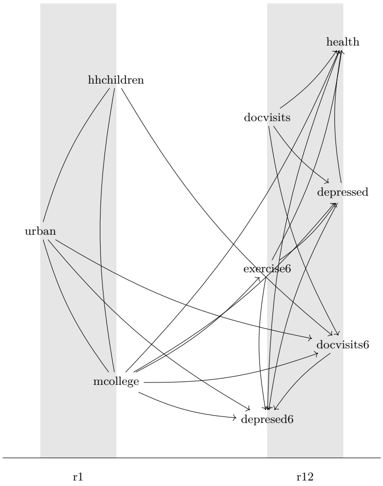


For details: Bang and Didelez [2023]

Finite sample data





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⇒ With background knowledge we do not necessarily get more informative graphs – but we expect fewer errors.

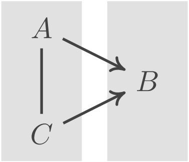
Moreover: Inconsistent independencies might result in conflicting edges – can be resolved by background knowledge.

Robustness

Fewer edges tests:

If $A \not\perp\!\!\!\perp C \mid \emptyset$ and $\mathrm{tier}(A) \leq \mathrm{tier}(C) < \mathrm{tier}(B)$,
then $A \not\perp\!\!\!\perp C \mid \{B\}$.

$\Rightarrow$ fewer type II errors (higher edge recall).



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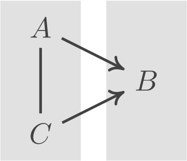

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Resolving conflicts:

Assume that both $A \to B \leftarrow C$ and $B \to C \leftarrow D$ then this might be resolved by background knowledge $\Rightarrow$ fewer conflicts.

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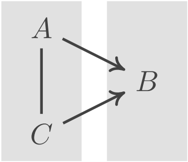

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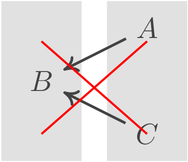
Fewer incorrect directed edges:

Suppose we incorrectly got $A \perp\!\!\!\perp C \mid \emptyset$
and $A \not\perp\!\!\!\perp C \mid \{B\}$,
but $\mathrm{tier}(B) < \mathrm{tier}(A) \leq \mathrm{tier}(C)$
$\Rightarrow$ here v-structure ruled out by the tiers.



Unobserved confounding

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Represent (pure) latent confounding using bidirected edges:

$$\begin{tikzcd} & L \arrow[dl, bend right] \arrow[dr, bend left] & \\ A & & B \end{tikzcd} \quad \Longrightarrow \quad \begin{tikzcd} A \arrow[rr, leftrightarrow, bend left] & & B \end{tikzcd}$$

Assume that all directed edges might be confounded:

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Task: Adapt the expert graph!

References

Christine W Bang and Vanessa Didelez. Do we become wiser with time? On causal equivalence with tiered background knowledge. Proceedings of the Thirty-Ninth Conference on Uncertainty in Artificial Intelligence, pages 119–129. PMLR, 2023.

Christine W Bang, Janine Witte, Ronja Foraita, and Vanessa Didelez. Efficient use of tiered background knowledge for causal discovery with cohort data. Working paper, 2024.

Emilija Perković, Markus Kalisch, and Marloes H Maathuis. Interpreting and using cpdags with background knowledge. In Proceedings of the 2017 Conference on Uncertainty in Artificial Intelligence (UAI2017), pages ID–120. AUAI Press, 2017.

Anne H Petersen, Merete Osler, and Claus T Ekstrøm. Data-driven model building for life-course epidemiology. American Journal of Epidemiology, 190(9):1898–1907, 2021.